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Probing supersymmetric flavor models with ε'/ε

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Abstract: We discuss the supersymmetric contribution to ε'/ε in various supersymmetric flavor models. We find that in alignment models the supersymmetric contribution could be significant while in heavy squark models it is expected to be small. The situation is particularly interesting in models that solve the flavor problems by either of the above mechanisms and the remaining CP problems by means of approximate CP, that is, all CP violating phases are small. In such models, the standard model contributions cannot account for ε'/ε and a failure of the supersymmetric contributions to do so would exclude the model. In models of alignment and approximate CP, the supersymmetric contributions can account for ε'/ε only if both the supersymmetric model parameters and the hadronic parameters assume rather extreme values. Such models are then strongly disfavored by the ε'/ε measurements. Models of heavy squarks and approximate CP are excluded.

Keywords: Supersymmetric Standard Model, CP violation, Kaon Physics.

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Contents

1. Introduction

The ε'/ε parameter, signifying direct CP violation, has now been measured with impressive accuracy [1]–[5]:

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (2.12 \pm 0.46) \times 10^{-3} \,. \tag{1.1}$$

The theoretical interpretation of this result suffers from large hadronic uncertainties. Within the Standard Model, the theoretically preferred range is somewhat lower than the experimental range of eq. (1.1) (for recent work, see [6]–[15] and references therein). Yet, if all the hadronic parameters are taking values at the extreme of their reasonable ranges, the experimental result can be accommodated.

While (1.1) does not provide unambiguous evidence for new physics, it is still useful in testing extensions of the Standard Model. Models where ε'_K is suppressed and/or ε_K enhanced are disfavored. Models that allow significant new contributions to ε'/ε may be favored if future improvement in the experimental measurement and, in particular, in the theoretical calculation will prove that the Standard Model fails to account for its large value. Investigations of the supersymmetric contributions to ε'/ε in view of the recent experimental results have been presented in refs. [16]–[20]. Most interestingly, in models where CP is an approximate symmetry of electroweak interactions, that is, all CP violating phases are small, it is clear already at present that the Standard Model cannot explain (1.1). These models should then provide new contributions to fully account for ε'/ε. A failure to do so would mean that the model is excluded. In this work we examine two such classes of supersymmetric flavor models. In the first class of models, a horizontal abelian symmetry solves the supersymmetric flavor problems by means of alignment and approximate CP solves the remaining CP problems. In the second class, the first two sfermion generations are heavy, thus solving most flavor problems, while mild alignment and approximate CP solve the remaining flavor and CP problems.

The plan of this paper goes as follows. In section 2, we present the possible supersymmetric sources of ε'/ε and discuss their uncertainties. In sections 3–5, we estimate the various contributions in models of abelian horizontal symmetries. In section 3, we study models where a horizontal symmetry explains the Yukawa hierarchy. In section 4, we require that, in addition, the symmetry solves the supersymmetric flavor problems. In section 5, we add the assumption of approximate CP. Models with heavy first two squark generations are discussed in section 6. We summarize our results in section 7.

2. Supersymmetric contributions to ε'/ε

In generic supersymmetric models, there are potentially many new contributions to ε'/ε from loop diagrams involving intermediate squarks and gluinos, charginos or neutralinos. If there is some degeneracy between squarks, then a convenient way to parametrize these contributions is by using the $(\delta^q_{MN})_{ij}$ parameters. In the basis where quark masses and gluino couplings are diagonal, the dimensionless $(\delta^q_{MN})_{ij}$ parameters stand for the ratio between $(M^2_{\tilde q})^{MN}_{ij}$, the (ij) entry $(i,j=1,2,3)$ in the mass-squared matrix for squarks ($M,N=L,R$ and $q=u,d$), and $\tilde m^2$, the average squark mass-squared. If there is no mass degeneracy among squarks, then these parameters can be related to the supersymmetric mixing angles. Defining K^d_L (K^d_R) to be the mixing matrix between left-handed (right-handed) down quarks and the scalar partners of left-handed (right-handed) down quarks, we have, e.g., $(\delta^d_{LL})_{12} \sim (K^d_L)_{12}$.

For supersymmetry to account for ε'/ε, at least one of the following conditions should be met [15]–[24]:

$$\begin{aligned}
\mathrm{Im}[(\delta^d_{LL})_{12}] &\sim \lambda \left(\frac{\tilde m}{500\,\mathrm{GeV}}\right)^2 ,\\
\mathrm{Im}[(\delta^d_{LR})_{12}] &\sim \lambda^7 \left(\frac{\tilde m}{500\,\mathrm{GeV}}\right) ,\\
\mathrm{Im}[(\delta^d_{LR})_{21}] &\sim \lambda^7 \left(\frac{\tilde m}{500\,\mathrm{GeV}}\right) ,
\end{aligned} \tag{2.1}$$

$$\begin{aligned}
\mathrm{Im}[(\delta^u_{LR})_{13}(\delta^u_{LR})^*_{23}] &\sim \lambda^2 ,\\
\mathrm{Im}[V_{td}(\delta^u_{LR})^*_{23}] &\sim \lambda^3 \left(\frac{M_2}{m_W}\right) ,\\
\mathrm{Im}[V^*_{ts}(\delta^u_{LR})_{13}] &\sim \lambda^3 \left(\frac{M_2}{m_W}\right) .
\end{aligned} \tag{2.2}$$

Here $\lambda = 0.2$ is a small parameter of order of the Cabibbo angle that is convenient to use in the context of flavor models.

Let us first discuss the three options of eq. (2.1). The first of these conditions violates constraints from Δm_K and ε_K. Therefore, independent of the supersymmetric flavor model, it cannot be satisfied. On the other hand, the requirements on $\mathrm{Im}[(\delta^d_{LR})_{12}]$ or $\mathrm{Im}[(\delta^d_{LR})_{21}]$ pose no phenomenological problem. Moreover, we will see in the next section that such values are not impossible within our theoretical framework. We therefore investigate more carefully the uncertainties in the corresponding condition. Using the following expression for the matrix element of the chromomagnetic operator [25]:

$$\langle(\pi\pi)_{I=0}|Q_g|K^0\rangle = \sqrt{\frac{3}{2}}\frac{11}{16\pi^2}\frac{\langle\bar{q}q\rangle}{F_\pi^3}m_\pi^2 B_G\,, \tag{2.3}$$

the expression for the Wilson coefficients at the SUSY scale given in [22] and LO QCD corrections, one can write the contribution of $\mathrm{Im}[(\delta^d_{LR})_{12}]$ and $\mathrm{Im}[(\delta^d_{LR})_{21}]$ to ε'/ε as follows [26]:

$$\begin{aligned} |\varepsilon'/\varepsilon| = 58\ B_G \left[\frac{\alpha_s(m_{\tilde g})}{\alpha_s(500\,\mathrm{GeV})}\right]^{23/21} \left(\frac{158\,\mathrm{MeV}}{m_s(m_c)+m_d(m_c)}\right) \times \\ \times \left(\frac{500\,\mathrm{GeV}}{m_{\tilde g}}\right) \left|\mathrm{Im}\left[(\delta^d_{LR})_{12} - (\delta^d_{LR})^*_{21}\right]\right|\,. \end{aligned} \tag{2.4}$$

Here the parameter B_G accounts for possible deviations of the hadronic matrix element in eq. (2.3) from the value obtained at lowest order in the chiral quark model. Other estimates based on the bag model [27] and on naive dimensional analysis [28] find $B_G = \mathcal{O}(1)$. None of these calculations is based on first principles and all of them are subject to uncontrolled theoretical uncertainties. Moreover, the agreement between these calculations is accidental in the sense that the "anomalous" m_π^2 suppression of the matrix element in eq. (2.3) that was found in the chiral quark model is not reproduced in the other two methods. In view of this fact and given the large uncertainties from higher order contributions within the chiral quark model, we use the conservative range $B_G \leq 5$ (see ref. [26] for a detailed derivation of eq. (2.4) and of the related hadronic uncertainties). Using

$$\begin{aligned} \varepsilon'/\varepsilon &\gtrsim\ 1\times 10^{-3}\,, \\ m_{\tilde g} &\gtrsim\ 150\,\mathrm{GeV}\,, \\ m_s(m_c) &\gtrsim\ 110\,\mathrm{MeV}\,, \end{aligned} \tag{2.5}$$

we get a lower bound:

$$\mathrm{Im}(\delta^d_{LR})_{12} \gtrsim 7\times 10^{-7}\,, \tag{2.6}$$

that is $\mathcal{O}(\lambda^9)$ or even $\mathcal{O}(\lambda^{10})$ if $\lambda \sim 0.24$. A similar bound applies to $\mathrm{Im}[(\delta^d_{LR})_{21}]$.

We now turn to the three options in eq. (2.2). These contributions to ε'/ε arise by inducing an effective Z_{ds} coupling, where [15]:

$$\mathcal{L}^Z_{\rm FC} = \frac{G_F}{\sqrt{2}}\frac{em_Z^2}{2\pi^2}\frac{\cos\theta_W}{\sin\theta_W} Z_{ds}\bar{s}\gamma_\mu(1-\gamma_5)dZ^\mu + {\rm h.c.}\,. \tag{2.7}$$

The contribution of such an effective coupling to ε'/ε is given by

$$\varepsilon'/\varepsilon = {\rm Im}\, Z_{ds}\left[1.2 - \left(\frac{158\,{\rm MeV}}{m_s(m_c)+m_d(m_c)}\right)^2 \left|r_Z^{(8)}\right| B_8^{(3/2)}\right], \tag{2.8}$$

where $B_8^{(3/2)}$ is the non-perturbative parameter describing the hadronic matrix element of the electroweak penguin operator and $\left|r_Z^{(8)}\right|$ is a calculable renormalization scheme independent parameter. We consider the following ranges [15]:

$$\begin{aligned} 6.5 \le \left|r_Z^{(8)}\right| &\le 8.5\,, \\ 0.6 \le B_8^{(3/2)} &\le 1.0\,. \end{aligned} \tag{2.9}$$

Then, for ${\rm Im}\, Z_{ds}$ to account for ε'/ε, we need

$$-\,{\rm Im}\, Z_{ds} \gtrsim \frac{\varepsilon'/\varepsilon}{16}. \tag{2.10}$$

To find the lower bound on ${\rm Im}[V_{td}(\delta^u_{LR})^*_{23}]$, we have performed a more careful analysis of its relation to Z_{ds}. We scanned the following range of supersymmetric parameters:

$$\begin{aligned} -300\,{\rm GeV} \le\ & \mu \ \le 300\,{\rm GeV}\,, \\ 100\,{\rm GeV} \le\ & M_2 \le 250\,{\rm GeV}\,, \\ 3M_2 \le\ & m_{\tilde{Q}} \le 5M_2\,, \\ 0.4m_{\tilde{Q}} \le\ & m_{\tilde{t}_R} \le m_{\tilde{Q}}\,, \end{aligned} \tag{2.11}$$

and discarded points in which chargino masses are $\le 90\,{\rm GeV}$. We found that

$${\rm Im}\, Z_{ds} \le 0.03\,{\rm Im}[V_{td}(\delta^u_{LR})^*_{23}]\,. \tag{2.12}$$

Together with eq. (2.10) and the lower bound on ε'/ε quoted in (2.5), we find then the following lower bound:

$${\rm Im}[V_{td}(\delta^u_{LR})^*_{23}] \gtrsim 2\times 10^{-3}\,, \tag{2.13}$$

that is $\mathcal{O}(\lambda^4)$. Bounds similar to (2.12) and (2.13) apply to ${\rm Im}[V^*_{ts}(\delta^u_{LR})_{13}]$. The lower bound on ${\rm Im}[(\delta^u_{LR})_{13}(\delta^u_{LR})^*_{23}]$ is stronger, of $\mathcal{O}(\lambda^3)$.

3. Abelian horizontal symmetries

Models of abelian horizontal symmetries are able to provide a natural explanation for the hierarchy in the quark and lepton flavor parameters [29]. The symmetry is broken by a small parameter λ which is usually taken to be of the order of the Cabibbo angle, $\lambda \sim 0.2$. The hierarchy in the flavor parameters is then a result of the selection rules related to the approximate horizontal symmetry. In the supersymmetric framework, holomorphy also plays a role in determining the Yukawa parameters [30].

For the sake of definiteness, we take the following order of magnitude estimates for the various quark mass ratios and mixing angles:

$$\begin{aligned} \frac{m_u}{m_c} &\sim \lambda^3\,, & \frac{m_c}{m_t} &\sim \lambda^4\,, & \frac{m_t}{\langle\phi_u\rangle} &\sim 1\,, \\ \frac{m_d}{m_s} &\sim \lambda^2\,, & \frac{m_s}{m_b} &\sim \lambda^2\,, & \frac{m_b}{m_t} &\sim \lambda^3\,, \\ |V_{us}| &\sim \lambda\,, & |V_{cb}| &\sim \lambda^2\,, & |V_{ub}| &\sim \lambda^3\,. \end{aligned} \tag{3.1}$$

Within models of a single horizontal U(1) symmetry, this set of parameters determines all the horizontal charges, leading to the following structure of the quark mass matrices:

$$M_u \sim \langle\phi_u\rangle \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle\phi_d\rangle \lambda^3 \tan\beta \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}. \tag{3.2}$$

A similar hierarchy appears also in the (LR) blocks of the corresponding squark mass-squared matrices:

$$(M^2_{\tilde{u}})^{LR}_{ij} \sim \tilde{m}(M_u)_{ij}\,, \qquad (M^2_{\tilde{d}})^{LR}_{ij} \sim \tilde{m}(M_d)_{ij}\,. \tag{3.3}$$

We emphasize that eq. (3.3) does *not* imply $(M^2_{\tilde{q}})^{LR} \propto M_q$. The coefficients of order one are independent and different in the respective entries. Only the parametric suppression is the same for the squarks and the quarks.

Eqs. (3.2) and (3.3) allow us to estimate the values of the δ_{LR} parameters of eq. (2.1) and (2.2). We get:

$$\begin{aligned} (\delta^d_{LR})_{12} &\sim \frac{m_s|V_{us}|}{\tilde{m}} \sim \lambda^6\,\frac{m_t}{\tilde{m}}\,, \\ (\delta^d_{LR})_{21} &\sim \frac{m_d}{|V_{us}|\;\tilde{m}} \sim \lambda^6\,\frac{m_t}{\tilde{m}}\,, \\ (\delta^u_{LR})_{13} &\sim \frac{m_t|V_{ub}|}{\tilde{m}} \sim \lambda^3\,\frac{m_t}{\tilde{m}}\,, \\ (\delta^u_{LR})_{23} &\sim \frac{m_t|V_{cb}|}{\tilde{m}} \sim \lambda^2\,\frac{m_t}{\tilde{m}}\,. \end{aligned} \tag{3.4}$$

Taking into account that $|V_{td}| \sim \lambda^3$ and $|V_{ts}| \sim \lambda^2$, we learn that the three options in eq. (2.2) are of order λ^{5-7}. We compare this to the requirements given in eq. (2.13) (and the discussion below this equation) and conclude that, in models of abelian horizontal symmetries, the contributions to Z_{ds} involving $\tilde{t}_R$ cannot account for ε'/ε.

On the other hand, $(\delta^d_{LR})_{12}$ and $(\delta^d_{LR})_{21}$ are large enough to allow for a supersymmetric explanation of ε'/ε [16]. This is the case even if the supersymmetric scale is not particularly low, say $\tilde{m} \sim 500\,\text{GeV}$. If the supersymmetric scale is lower, then a supersymmetric explanation of ε'/ε is possible even for small phases, $\phi_{CP} \sim \lambda^{2-4}$, if other relevant parameters take extreme values, as in eq. (2.6).

4. Alignment

It is possible to solve the supersymmetric flavor problems by the mechanism of alignment [31]–[34], whereby the mixing matrices for gaugino couplings have very small mixing angles. Alignment arises naturally in the framework of abelian horizontal symmetries. Simple models give supersymmetric mixing angles that are similar to the corresponding CKM mixing angles. However, for the mixing between the first two down squark generations, a much more precise alignment is needed, that is,

$$(K^d_L)_{12} \lesssim \lambda^2\,, \qquad (K^d_R)_{12} \lesssim \lambda^2\,, \qquad (K^d_L)_{12}(K^d_R)_{12} \lesssim \lambda^6\,. \tag{4.1}$$

Eq. (4.1) implies that some of the entries in M_d should be suppressed compared to their 'naive' values, given in eq. (3.2). In particular, the following constraints should be satisfied:

$$\begin{aligned} \frac{(M_d)_{12}}{(M_d)_{22}} &\lesssim \lambda^2\,, \\ \frac{(M_d)_{21}}{(M_d)_{22}} &\lesssim \lambda^2\,, \\ \frac{(M_d)_{12}(M_d)_{21}}{[(M_d)_{22}]^2} &\lesssim \lambda^6\,. \end{aligned} \tag{4.2}$$

Additional constraints apply to $(M_d)_{31}$ and $(M_d)_{32}$, but they are irrelevant to our study here.

To achieve the required suppression, one has to employ a more complicated abelian horizontal symmetry. The models of refs. [31]–[34] use $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetries. Then, it is possible to retain all the 'good' predictions of eq. (3.1) and, at the same time, have the Yukawa couplings that are relevant to (4.1) vanish due to holomorphy of the superpotential, that is, $(M_d)_{12} = 0$, $(M_d)_{21} = 0$, either $(M_d)_{13}$ or $(M_d)_{32} = 0$ and either $(M_d)_{31}$ or $(M_d)_{23} = 0$. The vanishing of these entries is exact in the basis where the horizontal charges are well defined. In this basis the kinetic terms are not canonically normalized. When we transform to a basis with canonical normalization of the kinetic terms, the holomorphic zeros are lifted [32]. However, they are suppressed by at least a factor of λ^2 compared to their naive values of eq. (3.2) [32, 35]. The suppression could be much stronger but it is always an even power of the breaking parameter.

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The entries in the LR-block of the down-squark mass-squared matrix, that is $(M^2_{\tilde d})^{LR}_{ij}$, are suppressed in a similar way to the corresponding entries in the down quark mass matrix, $(M_d)_{ij}$. Consequently, the alignment requirements (4.1) affect directly $(M^2_{\tilde d})^{LR}_{12}$ and $(M^2_{\tilde d})^{LR}_{21}$ that are relevant to ε'/ε. Independent of the details of the model, we find that in the framework of alignment, we have

$$\begin{aligned}(\delta^d_{LR})_{12} &\lesssim \frac{m_s|V_{us}|}{\tilde m}\lambda^2 \sim \lambda^8\,\frac{m_t}{\tilde m}\,,\\ (\delta^d_{LR})_{21} &\lesssim \frac{m_d}{|V_{us}|\;\tilde m}\lambda^2 \sim \lambda^8\,\frac{m_t}{\tilde m}\,.\end{aligned} \tag{4.3}$$

The values in eq. (4.3) should be compared with the phenomenological input of eq. (2.1). It is interesting that for central values of the hadronic parameters, the supersymmetric contributions to ε'/ε in models of alignment can naturally be of the required order of magnitude. For this to happen, the models have to satisfy two conditions:

(i) The alignment has to be minimal in the sense explained above, that is either $|(K^d_L)_{12}| \sim \lambda^3$ or $|(K^d_R)_{12}| \sim \lambda^3$ should hold.

(ii) The relevant phase is of order one.

We note that a situation where both $|(K^d_L)_{12}|$ and $|(K^d_R)_{12}|$ are of order λ^3 and with a phase of order one is forbidden since it will give a contribution to ε_K that is too large.

Our analysis here (and in the next section) applies also to the interesting mechanism suggested in ref. [19] . There, instead of the conventional $(\delta^d_{LR})_{12}$ considered by us, it was suggested that such an effective flavor- and chirality-changing mass insertion could be induced by a combination of flavor-changing chirality-conserving and flavor-conserving chirality-changing mass insertions, that is $(\delta^d_{LR})^{\rm ind}_{12} = (\delta^d_{LL})_{12}(\delta^d_{LR})_{22}$. Within our framework, a naive estimate would yield $(\delta^d_{LL})_{12} \sim |V_{us}|$ and $(\delta^d_{LR})_{22} \sim m_s/\tilde m$. Such a large $(\delta^d_{LL})_{12}$ is, however, excluded by the Δm_K constraint. With alignment, $(\delta^d_{LL})_{12}$ is suppressed by at least a factor of λ^2 while $(\delta^d_{LR})_{22}$ is unaffected. We learn that, in the framework of abelian horizontal symmetries, the estimates of eq. (4.3) applies also to $(\delta^d_{LR})^{\rm ind}_{12}$.

5. Approximate CP

The requirement that supersymmetric contributions to flavor changing CP violation, that is the ε_K parameter, and to flavor diagonal CP violation, such as the electric dipole moment of the neutron d_N, are not too large, puts severe constraints on the supersymmetric parameters (for a review, see [36]). For example, the ε_K constraints read

$$\sqrt{\mathrm{Im}[(\delta^d_{LL})^2_{12}]} \lesssim \lambda^3\,,$$
$$\sqrt{\mathrm{Im}[(\delta^d_{RR})^2_{12}]} \lesssim \lambda^3\,,$$
$$\sqrt{\mathrm{Im}[(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}]} \lesssim \lambda^5\,. \tag{5.1}$$

The third constraint is particularly strong. For example, for $|(\delta^d_{LL})_{12}| \sim |(\delta^d_{RR})_{12}| \sim \lambda^3$, the phase in their product needs to be smaller than $\mathcal{O}(\lambda^4)$. The d_N constraint generically requires that the flavor diagonal phases ϕ_A (related to the trilinear scalar couplings) and ϕ_B (related to bilinear $\phi_u\phi_d$ terms) are small,

$$\phi_A \lesssim 10^{-2}\,, \qquad \phi_B \lesssim 10^{-2}\,. \tag{5.2}$$

Approximate CP is then a well motivated option in supersymmetric models.

Before discussing models with alignment and approximate CP, we would like to make the following comments. It is possible to construct models of alignment with the following features [34]:

(i) The alignment is precise enough that it solves not only the supersymmetric flavor problems but also the supersymmetric ε_K problem.

(ii) The ϕ_A and ϕ_B phases are small enough to solve the supersymmetric d_N problem.

(iii) The Kobayashi-Maskawa phase is of order one, so that ε'/ε can be explained by Standard Model contributions.

The explicit models of ref. [34] where these features are realized are very constrained and almost unique. We learn that, on one hand, approximate CP is not a necessary ingredient in models of alignment but, on the other hand, it allows simpler and less constrained models of this type.

We focus then on models where all flavor problems are solved by alignment, but the CP problems are solved by approximate CP. The main point is that, independent of the details of the model, the CP violating phases in this framework are suppressed by even powers of the breaking parameter. The reason for that is as follows. In the framework of abelian horizontal symmetries, all contributions to a given term should carry the appropriate horizontal charge. If the CP violating contribution does not appear in leading order, then it should be suppressed by a neutral combination of the breaking parameter, such as $\lambda\lambda^*$. An odd power of the breaking parameter always carries a horizontal charge.

The conclusion is that, in models of approximate CP, the imaginary part of any $(\delta^q_{MN})_{ij}$ term is suppressed by, at least, a factor of λ^2 compared to the real part. In particular, we have

$$\begin{aligned}
\mathrm{Im}(\delta^d_{LR})_{12} &\lesssim \tfrac{m_s|V_{us}|}{\tilde m}\lambda^4 \sim \lambda^{10}\tfrac{m_t}{\tilde m}\,,\\
\mathrm{Im}(\delta^d_{LR})_{21} &\lesssim \tfrac{m_d}{|V_{us}|\ \tilde m}\lambda^4 \sim \lambda^{10}\tfrac{m_t}{\tilde m}\,.
\end{aligned} \tag{5.3}$$

These are rather low values. They are consistent with the experimental constraint of eq. (2.6) only if all the following conditions are simultaneously satisfied:

(i) The suppression of the relevant CP violating phases is 'minimal', that is $\mathcal{O}(\lambda^2)$.

(ii) The alignment of the first two down squark generations is 'minimal', that is $\mathcal{O}(\lambda^2)$.

(iii) The mass scale for the supersymmetric particles is low, $\tilde m \sim 150\,\mathrm{GeV}$.

(iv) The hadronic matrix element is larger than what hadronic models suggest, $B_G \sim 5$.

(v) The mass of the strange quark is at the lower side of the theoretically preferred range, $m_s(m_c) \sim 110\,\mathrm{MeV}$.

(vi) The value of ε'/ε is at the lower side of the experimentally allowed range.

While such a combination of conditions on both the supersymmetric models and the hadronic parameters is not very likely to be realized, it cannot be rigorously excluded either. We conclude that models that combine alignment and approximate CP are disfavored by the measurement of ε'/ε.

6. Heavy squarks

Most of the supersymmetric flavor and CP problems are solved if the masses of the first and second generation squarks $\tilde m_h$ are larger than the other soft masses, $\tilde m_l$: $\tilde m_h^2 \sim 100\,\tilde m_l^2$ [37]–[39]. This does not necessarily lead to naturalness problems, since these two generations are almost decoupled from the Higgs sector. Explicit models are presented in [39]–[51]. We here follow mainly the discussion in [40, 41] but our main points are of more general validity.

The main feature of these models that is relevant to our discussion of ε'/ε is that the supersymmetric breaking scale that appears in the A terms is (at most) of the order of the electroweak scale while the scale that characterizes the average mass of the down and strange squark masses is $\tilde m_h$. Consequently, the δ_{LR} parameters are strongly suppressed:

$$(\delta^q_{LR})_{ij} \lesssim \frac{m_Z (m_q)_{ij}}{\tilde m_h^2} \sim 10^{-4}\frac{(m_q)_{ij}}{m_Z}\,. \tag{6.1}$$

In this expression, $(m_q)_{ij}$ is related to the flavor structure of the model, which is not always defined in the above models. In any case, it is $\lesssim m_b$ (m_t) in the down (up) sector. Independent of the details of the model we have then

$$\begin{aligned}(\delta^d_{LR})_{ij} &\lesssim 10^{-4}\left(\tfrac{m_b}{m_Z}\right) \sim 5\times 10^{-6}\,,\\ (\delta^u_{LR})_{ij} &\lesssim 10^{-4}\left(\tfrac{m_t}{m_Z}\right) \sim 2\times 10^{-4}\,. \end{aligned} \tag{6.2}$$

Comparing these upper bounds to eqs. (2.6) and (2.13), we learn that, similarly to the models of abelian horizontal symmetries, only $(\delta^d_{LR})_{12}$ and $(\delta^d_{LR})_{21}$ can account for ε'/ε.

The suppression from the large $\tilde{d}$ and $\tilde{s}$ masses is not enough, however, to satisfy the Δm_K constraint. A mild alignment, $(K^d_L)_{12} \sim \lambda$, is required. Therefore, the upper bound on $(\delta^d_{LR})_{12}$ and $(\delta^d_{LR})_{21}$ is actually of $\mathcal{O}(10^{-6})$. If we make the further plausible assumption that this alignment reflects a flavor structure that is similar to that of the corresponding quark mass matrix, we have

$$(\delta^d_{LR})_{12} \lesssim 10^{-4}\frac{m_s|V_{us}|}{m_Z} \sim 3\times 10^{-8}\,. \tag{6.3}$$

We learn that in the likely situation that $(m_d)_{12} \sim m_s|V_{us}|$, $(\delta^d_{LR})_{12}$ is already below the value where it could potentially give a significant contribution to ε'/ε. However, if the flavor structure is such that $(m_d)_{12} \sim m_b|V_{us}|$, a significant contribution is not excluded. We emphasize, however, that there is no flavor model that predicts such a large value.

The combination of heavy squark masses and alignment of order of the Cabibbo angle is still not enough to satisfy the ε_K constraint. This constraint is satisfied if the CP violating supersymmetric phases in the down and strange couplings are less than $\mathcal{O}(1/30)$ [41]. Then, very likely, this phase also suppresses $\mathrm{Im}[(\delta^d_{LR})_{12}]$, that is,

$$\mathrm{Im}[(\delta^d_{LR})_{12}] \lesssim \frac{m_Z m_s|V_{us}|\phi_{\mathrm{CP}}}{\tilde{m}_h^2} \sim 10^{-9}\,. \tag{6.4}$$

With a different flavor structure, we can imagine an enhancement of this contribution by a factor of order, at most, m_b/m_s, that is to 3×10^{-8}. In any case, this contribution is smaller than the lower bound in (2.6) and, therefore, cannot account for ε'/ε.

If we make the final assumption, that the smallness of CP violating phases in the down and strange squark sector is not accidental and special to this sector but rather reflects an approximate CP symmetry in all interactions, that is all CP violating phases are small, then the Standard Model contributions are also too small and this class of models is excluded. We would like to emphasize, however, the following two points:

(i) The motivation for approximate CP is weaker with heavy squarks than it is with alignment. The reason is that the heavy squarks suppress to a satisfactory level the supersymmetric contributions to electric dipole moments even for (flavor-diagonal) CP violating phases of $\mathcal{O}(1)$. In contrast, alignment has no effect on flavor-diagonal CP violation.

(ii) The ε_K constraint applies to $\mathrm{Im}\{[(\delta^d_{LL})_{12}]^2\}$. It could be satisfied, therefore, not only by a very small phase (that is, $\mathrm{Im}[(\delta^d_{LL})_{12}] \ll |(\delta^d_{LL})_{12}|$) but also by a phase that is very close to $\pi/2$ (that is, $\mathrm{Re}[(\delta^d_{LL})_{12}] \ll |(\delta^d_{LL})_{12}|$). Then our discussion here of approximate CP is irrelevant. We note, however, that we know of no model which predicts such a near-maximal phase.

7. Conclusions

Supersymmetric models suffer from two problems related to CP violation. First, the ϕ_A and ϕ_B phases give an electric dipole moment of a neutron that is two orders of magnitude above the experimental bound, unless the phases are small or squarks of the first two generations heavy. Second, in models without universality, phases in the mixing matrices for gaugino couplings to quarks and squarks give a value to ε_K that is higher than the experimental value. There are three known ways in this type of models to suppress these contributions:

(i) Horizontal non-abelian symmetries lead to approximate degeneracy between the first two squark generations;

(ii) Horizontal abelian symmetries lead to suppression of the mixing angles by alignment of the mass matrices;

(iii) The first two squark generations could be heavy.

The models with horizontal (abelian or non-abelian) symmetries do not solve in general the d_N problem. Whether the ε_K problem is solved depends on the details of the model. If, in addition to employing one of these mechanisms to suppress flavor violation, there is also approximate CP to suppress CP violation, then both the d_N problem and the ε_K problem are solved.

The models with heavy squarks do solve the d_N problem but, in general, neither the Δm_K nor the ε_K constraints are satisfied. Alignment of order of the Cabibbo angle can solve the first and small CP violating phases the second problem.

If there is an approximate CP in nature, then the Standard Model with $\delta_{\rm KM} \ll 1$ cannot account for the measured value of ε'/ε. If the supersymmetric models with approximate CP are to be viable then they have to provide a supersymmetric mechanism to explain ε'/ε.

We have first examined this problem in the framework of alignment and reached the following conclusions:

(i) Models that combine alignment (to solve the supersymmetric flavor problem) and approximate CP (to solve the supersymmetric CP problems) are strongly disfavored. Only if several model parameters as well as several hadronic parameters take rather extreme values, the models are viable.

(ii) Models of abelian horizontal symmetries and approximate CP where the flavor problems are solved by a mechanism different from alignment can account for ε'/ε.

(iii) Models of alignment without approximate CP are likely to give a significant supersymmetric contribution to ε'/ε, in addition to the Standard Model contribution.

We then examined the problem in the framework of heavy squarks and reached the following conclusions:

(i) Models that combine heavy first two squark generations with alignment of order of the Cabibbo angle to solve the supersymmetric flavor problem and employ approximate CP to solve the supersymmetric ε_K problem are excluded.

(ii) Models of heavy squarks without approximate CP are still unlikely to give a significant supersymmetric contribution to ε'/ε.

In the near future, we expect first measurements of various CP asymmetries in B decays, such as $B \to \psi K_S$ or $B^\pm \to \pi^0 K^\pm$. If these asymmetries are measured to be of order one, it will support the Standard Model picture, that the CP violation that has been measured in the neutral K decays is small because it is screened by small mixing angles, while the idea that CP violation is small because all CP violating phases are small will be excluded. It is interesting, however, that various specific models that realize the latter idea, such as those discussed in this work, can already be excluded by the measurement of a tiny CP violating effect, $\varepsilon'_K \sim 5 \times 10^{-6}$.

After the completion of this work, a new lattice result appeared [52]. It finds that the value of the hadronic matrix element that is relevant to the standard model contribution to ε'/ε is larger by about one order of magnitude than (and of opposite sign to) its value in the vacuum insertion approximation. If this surprising result is confirmed, then the framework of supersymmetry with alignment and with approximate CP becomes attractive: ε_K is naturally accounted for by supersymmetric contributions with a small phase in $(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}$ while ε'/ε is naturally accounted for by the standard model contributions with a small $\delta_{\rm KM}$ [53]. In particular, model I of ref. [35] is a viable example of this idea.

Another interesting work that appeared after the completion of our work [54] shows that gluino box diagrams can account for ε'/ε if isospin breaking from $\tilde{u}_R - \tilde{d}_R$ mass splitting is large enough. While the framework of abelian horizontal symmetries allows for a large mass splitting, we found that in all existing models of alignment the supersymmetric contributions to the relevant operators are small. There are generic reasons for this suppression, but it is still an open question [55] whether one could construct specific models of alignment that would account for ε'/ε via the mechanism of ref. [54].

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